



MATHEMATICAL ASSOCIATION OF AMERICA

MAA

American Mathematics Competitions

28th Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 13, 2012



INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

2012
AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 13, 2012

****ADMINISTRATION ON AN EARLIER DATE
WILL DISQUALIFY YOUR SCHOOL'S RESULTS****

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 13, 2012. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 13.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

The American Mathematics Competitions

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1. Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

(A) 6 (B) $6\frac{2}{3}$ (C) $7\frac{1}{2}$ (D) 8 (E) 9



2. In the county of East Westmore, statisticians estimate there is a baby born every 8 hours and a death every day. To the nearest hundred, how many people are added to the population of East Westmore each year?

(A) 600 (B) 700 (C) 800 (D) 900 (E) 1000

3. On February 13 *The Oshkosh Northwestern* listed the length of daylight as 10 hours and 24 minutes, the sunrise as 6:57 AM, and the sunset as 8:15 PM. The length of daylight and sunrise were correct, but the sunset was wrong. When did the sun really set?

(A) 5:10 PM (B) 5:21 PM (C) 5:41 PM (D) 5:57 PM (E) 6:03 PM

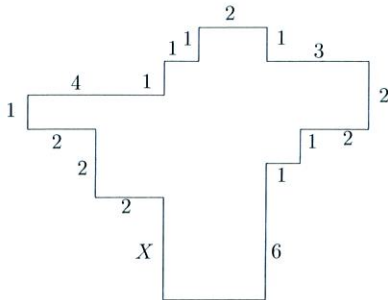


4. Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

(A) $\frac{1}{24}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$



5. In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is X , in centimeters?



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
6. A rectangular photograph is placed in a frame that forms a border two inches wide on all sides of the photograph. The photograph measures 8 inches high and 10 inches wide. What is the area of the border, in square inches?
- (A) 36 (B) 40 (C) 64 (D) 72 (E) 88

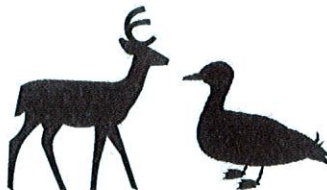


7. Isabella must take four 100-point tests in her math class. Her goal is to achieve an average grade of at least 95 on the tests. Her first two test scores were 97 and 91. After seeing her score on the third test, she realized that she could still reach her goal. What is the lowest possible score she could have made on the third test?
- (A) 90 (B) 92 (C) 95 (D) 96 (E) 97
8. A shop advertises that everything is “half price in today’s sale.” In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage discount off the original price?
- (A) 10 (B) 33 (C) 40 (D) 60 (E) 70



9. The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

(A) 61 (B) 122 (C) 139 (D) 150 (E) 161



10. How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 12

11. The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, x are all equal. What is the value of x ?

(A) 5 (B) 6 (C) 7 (D) 11 (E) 12

12. What is the units digit of 13^{2012} ?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

13. Jamar bought some pencils costing more than a penny each at the school bookstore and paid \$1.43. Sharona bought some of the same pencils and paid \$1.87. How many more pencils did Sharona buy than Jamar?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6



14. In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?

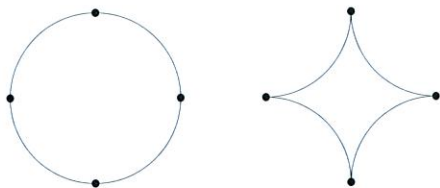
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

15. The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?
- (A) 40 and 50 (B) 51 and 55 (C) 56 and 60 (D) 61 and 65
(E) 66 and 99
16. Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?
- (A) 76531 (B) 86724 (C) 87431 (D) 96240 (E) 97403
17. A square with an integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
18. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?
- (A) 3127 (B) 3133 (C) 3137 (D) 3139 (E) 3149
19. In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?
- (A) 6 (B) 8 (C) 9 (D) 10 (E) 18

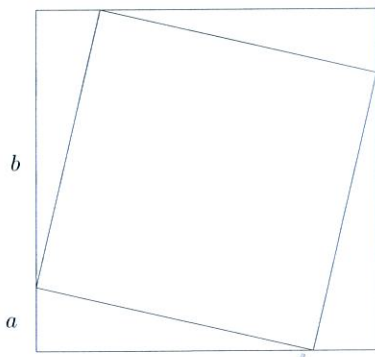


20. What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?
- (A) $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$ (B) $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$ (C) $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$
(D) $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$ (E) $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$
21. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?
- (A) $5\sqrt{2}$ (B) 10 (C) $10\sqrt{2}$ (D) 50 (E) $50\sqrt{2}$

22. Let R be a set of nine distinct integers. Six of the elements of the set are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R ?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
23. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?
- (A) 4 (B) 5 (C) 6 (D) $4\sqrt{3}$ (E) $6\sqrt{3}$
24. A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



- (A) $\frac{4 - \pi}{\pi}$ (B) $\frac{1}{\pi}$ (C) $\frac{\sqrt{2}}{\pi}$ (D) $\frac{\pi - 1}{\pi}$ (E) $\frac{3}{\pi}$
25. A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length a and the other of length b . What is the value of ab ?



- (A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) 4



SOLUTIONS

Your School Manager will be sent at least one copy of the 2012 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

Dr. Margie Raub Hunt, AMC 8 Chair
2169 Madero Dr., The Villages, FL 32159

Comments about administrative arrangements should be addressed to:

MAA American Mathematics Competitions / amcinfo@maa.org
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2013 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions

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PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:
amc.maa.org